

QCD SUM RULES STUDY OF Ξ_c AND Ξ_b BARYONS

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We use QCD sum rules to study the masses of the baryons Ξ_c and Ξ_b . We work with a current where the strange and the light quarks are in a relative spin zero, at leading order in α_s . We consider the contributions of condensates up to dimension six. For Ξ_b we get $m_{\Xi_b} = (5.75 \pm 0.25)$ GeV, and for Ξ_c we get $m_{\Xi_c} = (2.5 \pm 0.2)$ GeV, both in excellent agreement with the experimental values. We also make predictions to the state $\Omega_b(ssb)$ obtaining $m_{\Omega_b} = (5.82 \pm 0.23)$ GeV.

PACS numbers: 11.55.Hx, 12.38.Lg, 12.39.-x

The recent observation of the Ξ_b^- baryon by D0 and CDF collaborations [1, 2] with mass in agreement with the prediction in ref. [3], has stimulated us to use the QCD sum rules (QCDSR) [4] to evaluate the mass of this state. Previous QCD sum rule calculations for beauty baryons have been done before for the Λ_b , Σ_b , Σ_b^* and for double beauty baryons [5] but not for $\Xi_b^-(dsb)$. Since in the QCDSR approach, hadronic masses are related with the vacuum condensates, the use of this method to evaluate hadronic masses is an important step in the understanding of the dynamical nature of these masses.

Here we follow ref. [3] and assume that the strange and light (sq) quarks in Ξ_b are in a relative spin zero state. Therefore, the most general (low dimension) current which interpolates the Ξ_b operator can be constructed from a combination between the two currents, formed with scalar and pseudoscalar diquarks:

$$\eta_Q = \epsilon_{abc} [(q_a^T C \gamma_5 s_b) + t(q_a^T C s_b) \gamma_5] Q_c, \quad (1)$$

where a, b, c are color indices, C is the charge conjugation matrix, Q denotes the heavy quark and t is the mixing parameter between the two currents. Of course the above interpolating field can also be used to study the Ξ_c (qsc) baryon.

The QCDSR is constructed from the two-point correlation function

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T [\eta_Q(x) \bar{\eta}_Q(0)] | 0 \rangle. \quad (2)$$

Lorentz covariance, parity and time reversal imply that the two-point correlation function in Eq. (2) has the form

$$\Pi(q) = \Pi_1(q^2) + \not{q} \Pi_2(q^2). \quad (3)$$

A sum rule for each invariant function Π_1 and Π_2 , in Eq.(3) can be obtained.

The calculation of the phenomenological side at the hadron level proceeds by writing a dispersion relation to each one of the invariant functions in Eq. (3):

$$\Pi_i^{phen}(q^2) = - \int ds \frac{\rho_i(s)}{q^2 - s + i\epsilon} + \dots, \quad (4)$$

where ρ_i is the spectral density and the dots represent subtraction terms. The spectral density is described, as usual, as a single sharp pole representing the lowest resonance plus a smooth continuum representing higher mass states:

$$\begin{aligned} \rho_1(s) &= \lambda^2 m_{\Xi_Q} \delta(s - m_{\Xi_Q}^2) + \rho_1^{cont}(s), \\ \rho_2(s) &= \lambda^2 \delta(s - m_{\Xi_Q}^2) + \rho_2^{cont}(s), \end{aligned} \quad (5)$$

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where λ^2 gives the coupling of the current with the low mass hadron of interest. For simplicity, it is assumed that the continuum contribution to the spectral density, $\rho_i^{cont}(s)$ in Eq. (5), vanishes below a certain continuum threshold s_0 . Above this threshold, it is assumed to be given by the result obtained with the OPE. Therefore, one uses the ansatz [6]

$$\rho_i^{cont}(s) = \rho_i^{OPE}(s)\Theta(s - s_0) , \quad (6)$$

with

$$\rho_i^{OPE}(s) = \frac{1}{\pi} \text{Im}[\Pi_i^{OPE}(s)] . \quad (7)$$

On the OPE side, we work at leading order in α_s and consider the contributions of condensates up to dimension six. We keep the terms which are linear in the strange-quark mass m_s . We use the momentum space expression for the charm quark propagator, while the light-quark part of the correlation function is calculated in the coordinate-space. After making a Borel transform of both sides, and transferring the continuum contribution to the OPE side, the sum rules for Ξ_Q baryon, up to dimension-six condensates can be written as:

$$\lambda^2 m_{\Xi_Q} e^{-m_{\Xi_Q}^2/M^2} = \int_{m_Q^2}^{s_0} ds e^{-s/M^2} \rho_1^{OPE}(s) + \Pi_1(M^2) , \quad (8)$$

$$\lambda^2 e^{-m_{\Xi_Q}^2/M^2} = \int_{m_Q^2}^{s_0} ds e^{-s/M^2} \rho_2^{OPE}(s) + \Pi_2(M^2) , \quad (9)$$

where

$$\begin{aligned} \rho_i^{OPE}(s) &= \rho_i^{pert}(s) + \rho_i^{\langle \bar{q}q \rangle}(s) + \rho_i^{\langle G^2 \rangle}(s) , \\ \Pi_i(M^2) &= \Pi_i^{\langle \bar{q}g\sigma.Gq \rangle}(M^2) + \Pi_i^{\langle \bar{q}q \rangle^2}(M^2) . \end{aligned} \quad (10)$$

In the structure 1 we get

$$\begin{aligned} \rho_1^{pert}(s) &= \frac{(1-t^2)m_Q^5}{2^7\pi^4} \left[(1-x) \left(\frac{1}{x^2} + \frac{10}{x} + 1 \right) + 6 \left(1 + \frac{1}{x} \right) \ln x \right] , \\ \rho_1^{\langle \bar{q}q \rangle}(s) &= -\frac{m_Q m_s}{2^3\pi^2} (1-x) \left[(1+t^2) \langle \bar{q}q \rangle - \frac{(1-t^2)\mathbb{B}}{2} \right] , \\ \rho_1^{\langle G^2 \rangle}(s) &= \frac{(1-t^2)m_Q \langle g^2 G^2 \rangle}{2^9 3\pi^4} \left[(1-x) \left(7 + \frac{2}{x} \right) + 6 \ln x \right] , \\ \Pi_1^{\langle \bar{q}g\sigma.Gq \rangle}(M^2) &= \frac{m_Q m_s}{2^5\pi^2} \left[\frac{(1-t^2) \langle \bar{s}g\sigma.Gs \rangle}{6} e^{-m_Q^2/M^2} - (1+t^2) \langle \bar{q}g\sigma.Gq \rangle \left(e^{-m_Q^2/M^2} - \int_0^1 d\alpha e^{\frac{-m_Q^2}{(1-\alpha)M^2}} \right) \right] , \\ \Pi_1^{\langle \bar{q}q \rangle^2}(M^2) &= \frac{m_Q \langle \bar{q}q \rangle \mathbb{B}}{6} (1+t^2) e^{-m_Q^2/M^2} , \end{aligned} \quad (11)$$

where $x = m_Q^2/s$. In the structure \not{q} we get

$$\begin{aligned} \rho_2^{pert}(s) &= \frac{(1+t^2)m_Q^4}{2^9\pi^4} \left[(1-x^2) \left(\frac{1}{x^2} - \frac{8}{x} + 1 \right) - 12 \ln x \right] , \\ \rho_2^{\langle \bar{q}q \rangle}(s) &= \frac{m_s}{2^4\pi^2} (1-x^2) \left[-(1-t^2) \langle \bar{q}q \rangle + \frac{(1+t^2)\mathbb{B}}{2} \right] , \\ \rho_2^{\langle G^2 \rangle}(s) &= \frac{(1+t^2) \langle g^2 G^2 \rangle}{2^{10} 3\pi^4} (1-x)(1+5x) , \\ \Pi_2^{\langle \bar{q}g\sigma.Gq \rangle}(M^2) &= \frac{m_s}{2^5\pi^2} \left[\frac{(1+t^2) \langle \bar{s}g\sigma.Gs \rangle}{6} e^{-m_Q^2/M^2} + (1-t^2) \langle \bar{q}g\sigma.Gq \rangle \left(e^{-m_Q^2/M^2} + \int_0^1 d\alpha (1-\alpha) e^{\frac{-m_Q^2}{(1-\alpha)M^2}} \right) \right] , \\ \Pi_2^{\langle \bar{q}q \rangle^2}(M^2) &= \frac{\langle \bar{q}q \rangle \mathbb{B}}{6} (1-t^2) e^{-m_Q^2/M^2} . \end{aligned} \quad (12)$$

The contribution of dimension-six condensates $\langle g^3 G^3 \rangle$ is neglected, since it is assumed to be suppressed by the loop factor $1/16\pi^2$.

In the numerical analysis of the sum rules, the values used for the quark masses and condensates are (see e.g. [4, 5]): $m_s = (0.10 \pm 0.03)$ GeV, $m_c(m_c) = (1.23 \pm 0.05)$ GeV, $m_b(m_b) = (4.24 \pm 0.06)$ GeV, $\langle \bar{q}q \rangle = -(0.23 \pm 0.03)^3$ GeV³, $\beta = 0.8 \langle \bar{q}q \rangle$, $\langle \bar{q}g\sigma.Gq \rangle = m_0^2 \langle \bar{q}q \rangle$ with $m_0^2 = 0.8$ GeV², $\langle g^2 G^2 \rangle = 0.88$ GeV⁴.

We start with the charmed baryon Ξ_c . We evaluate the sum rules in the range $1.5 \leq M^2 \leq 3.0$ GeV² for s_0 in the range: $3.0 \leq \sqrt{s_0} \leq 3.2$ GeV. In Fig. 1 we show the contribution of each term in Eq. (12) to

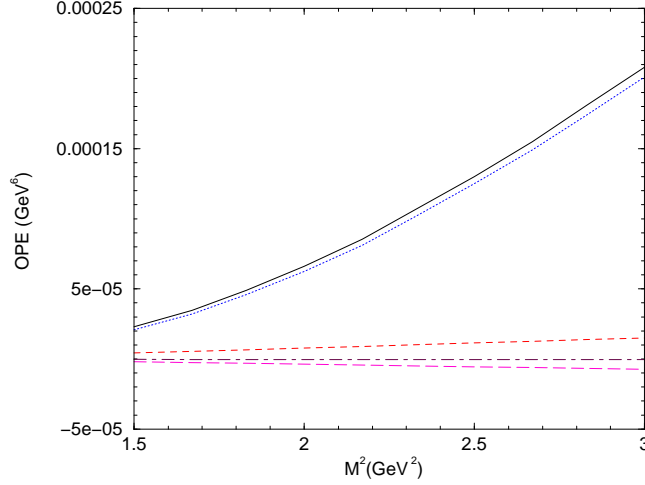


FIG. 1: The OPE convergence for the sum rule Eq.(9) for Ξ_c , using $\sqrt{s_0} = 3.1$ GeV and $t = 1$. The dotted, long-dashed, dashed and dot-dashed lines give, respectively, the perturbative, quark condensate, gluon condensate and mixed condensate contributions. The solid line gives the total OPE contribution to the sum rule.

the sum rule in Eq. (9), for $t = 1$ and $\sqrt{s_0} = 3.1$ GeV. We see that we get an excellent OPE convergence. For $t = 1$ the four-quark condensate contribution to the sum rule Eq. (9) vanishes. For other values of t , although the four-quark condensate contribution is bigger than the contribution of the other condensates, it is still much smaller than the perturbative contribution and, therefore, it does not spoil the convergence of the sum rule, as can be seen in Fig. 2. Therefore we conclude that the convergence of the sum rule

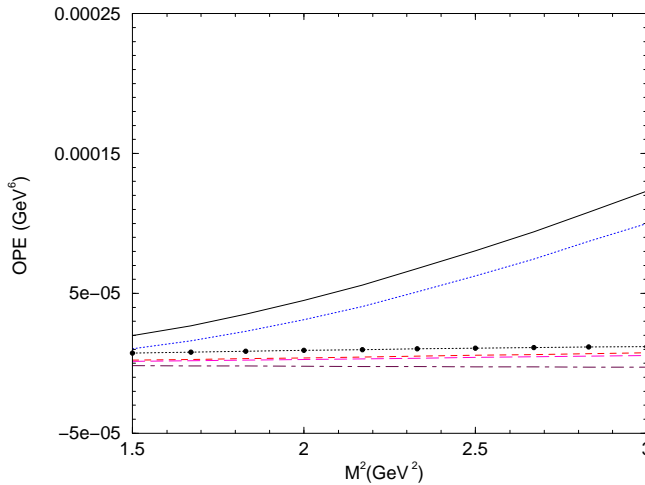


FIG. 2: Same as Fig. 1 for $t = 0$. The solid line with dots gives the four-quark condensate contribution

in the \not{q} structure in Eq. (9), is good for any value of the mixing parameter t (see Eq.(1)) in the range $0 \leq t \leq 1$. This is not the case of the sum rule in Eq. (8) (structure 1), since for $t = 1$ the perturbative and gluon condensate contributions vanish. As a matter of fact, if we try to obtain the mass of the Ξ_c baryon by dividing Eq. (8) by Eq. (9), we only get values compatible with the experimental mass for $t \sim 0$.

Therefore, in this work we will use only the sum rule in Eq. (9). To obtain the mass of the baryon we take the derivative of Eq. (9) with respect to $1/M^2$, divide the result by Eq. (9) and obtain:

$$m_{\Xi_Q}^2 = \frac{\int_{m_Q^2}^{s_0} ds e^{-s/M^2} s \rho_2^{OPE}(s) - (d\Pi_2/dM^{-2})}{\int_{m_Q^2}^{s_0} ds e^{-s/M^2} \rho_2^{OPE}(s) + \Pi_2(M^2)}. \quad (13)$$

We get an upper limit constraint for M^2 by imposing the rigorous constraint that the QCD continuum contribution should be smaller than the pole contribution. The maximum value of M^2 for which this constraint is satisfied depends on the value of s_0 and t . The comparison between pole and continuum contributions for $\sqrt{s_0} = 3.1$ GeV and $t = 1$ is shown in Fig. 3. The same analysis for the other values of

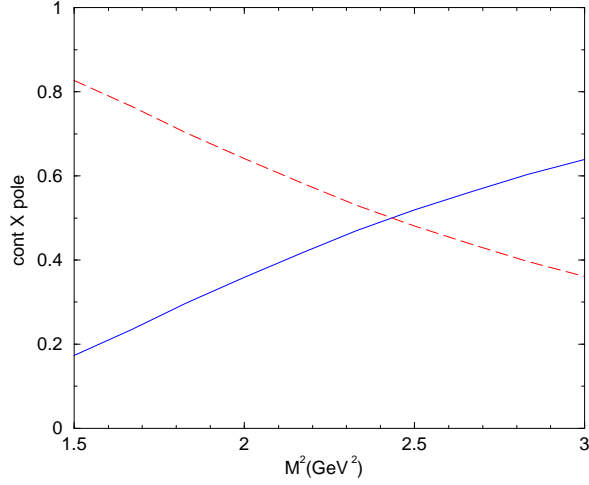


FIG. 3: The dashed line shows the relative pole contribution (the pole contribution divided by the total, pole plus continuum, contribution) and the solid line shows the relative continuum contribution for $\sqrt{s_0} = 3.1$ GeV and $t = 1$.

the continuum threshold and $t = 1$ gives $M^2 \leq 2.3$ GeV² for $\sqrt{s_0} = 3.0$ GeV and $M^2 \leq 2.65$ GeV² for $\sqrt{s_0} = 3.2$ GeV. We get similar results for other values of t , for example for $\sqrt{s_0} = 3.1$ GeV and $t = 0$ we get $M^2 \leq 2.6$ GeV².

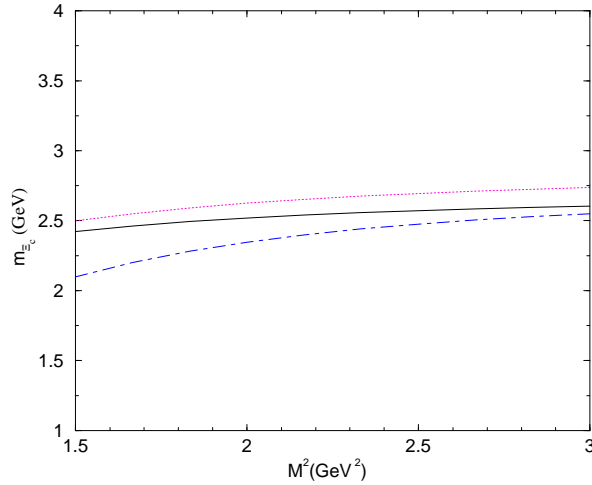


FIG. 4: The Ξ_c baryon mass as a function of the Borel parameter (M^2) for different values of the continuum threshold and the current mixing parameter: $t = 1$, $\sqrt{s_0} = 3.2$ GeV (solid line); $t = 1$, $\sqrt{s_0} = 3.0$ GeV (dotted line); $t = 0$, $\sqrt{s_0} = 3.1$ GeV (dot-dashed line).

In Fig. 4, we show the Ξ_c baryon mass obtained from Eq. (13), in the relevant sum rules window for

different values of $\sqrt{s_0}$ and t . From Fig. 4 we see that the results are more stable, as a function of M^2 , for $t = 1$ than for $t = 0$. Therefore, we will use $t = 1$ to estimate the mass of the particle. It is very interesting to notice that the result for the mass obtained with $t = 0$ is very similar to the one obtained dividing Eq. (8) by Eq. (9) using also $t = 0$.

We found that our results are not very sensitive to the value of the charm quark mass, neither to the value of the condensates. The most important source of uncertainty is the value of the continuum threshold and the Borel interval. Using the QCD parameters given above, the QCDSR result for the Ξ_c baryon mass is:

$$m_{\Xi_c} = (2.5 \pm 0.2) \text{ GeV}, \quad (14)$$

in a very good agreement with the experimental value $m_{\Xi_c}^{exp} = (2.4710 \pm 0.0004) \text{ GeV}$ [7].

In the case of the beauty baryon Ξ_b , using consistently the perturbative \overline{MS} -mass $m_b(m_b) = (4.24 \pm 0.6) \text{ GeV}$, $t = 1$ and the continuum threshold in the range $6.3 \leq \sqrt{s_0} \leq 6.5 \text{ GeV}$, we find a good OPE convergence for $M^2 > 4.0 \text{ GeV}^2$. We also find that the pole contribution is bigger than the continuum contribution for $M^2 < 5.2 \text{ GeV}^2$ for $\sqrt{s_0} < 6.3 \text{ GeV}$, and for $M^2 < 5.7 \text{ GeV}^2$ for $\sqrt{s_0} = 6.5 \text{ GeV}$.

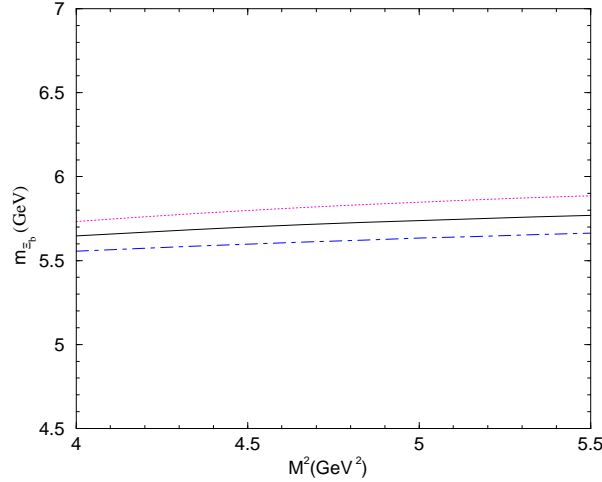


FIG. 5: The Ξ_b baryon mass as a function of the Borel parameter (M^2) for different values of the continuum threshold and the current mixing parameter: $t = 1$, $\sqrt{s_0} = 6.3 \text{ GeV}$ (solid line); $t = 1$, $\sqrt{s_0} = 6.5 \text{ GeV}$ (dotted line); $t = 0$, $\sqrt{s_0} = 6.4 \text{ GeV}$ (dot-dashed line).

From Fig. 5, where we show the Ξ_b baryon mass, we see that the results are very stable as a function of M^2 in the allowed Borel region. For completeness we also show, in Fig. 5, the results obtained using $t = 0$ and $\sqrt{s_0} = 6.4 \text{ GeV}$. In this case the results are very stable for all values of t in the range $0 \leq t \leq 1$. Therefore, we will also use different values of t to estimate the uncertainties in the result. Taking into account the variation of M^2 and varying s_0 , t and m_b in the regions indicated above, we arrive at the result:

$$m_{\Xi_b} = (5.75 \pm 0.25) \text{ GeV}, \quad (15)$$

also in a very good agreement with the predictions in refs. [3] and [8], and with the experimental results in ref. [1]: $m_{\Xi_b}^{D^0} = (5.774 \pm 0.013) \text{ GeV}$, and in ref. [2]: $m_{\Xi_b}^{CDF} = (5.7929 \pm 0.0024) \text{ GeV}$.

We have presented a QCDSR analysis of the two-point functions of the $\Xi_Q(qsQ)$ baryons. We find that the sum rules results for the masses of Ξ_c and Ξ_b are compatible with the experimental values and with the predictions in refs. [3, 8], in the case of Ξ_b . These results for Ξ_b are summarized in Table I.

Table I: Results for the Ξ_b mass.

$m_{\Xi_b} \text{ (GeV)}$	ref.
$5.774 \pm .011 \pm 0.015$	D0 [1]
$5.7929 \pm 0.0024 \pm 0.0017$	CDF [2]
5.795 ± 0.005	[3]
5.8057 ± 0.0081	[8]
5.75 ± 0.25	this work

It is important to notice that while the calculation based on modeling hyperfine interaction [3, 8] needs, as inputs, the masses of others heavy baryons, in our calculations the masses are extract using only information about the QCD parameters as quark masses and condensates. In the case of the heavy baryons Ξ_c and Ξ_b , in particular, their masses are determined basicaly by the first term in the sum rules Eqs. (8) and (9) and the condensates are not very important.

We have tested two different choices of currents. While the results for Ξ_c are sensitive to this choice, in the case of Ξ_b the results are very stable for the mixing parameter in the range $0 \leq t \leq 1$.

It is not possible to generalize directly our results to the baryons $\Omega_c(ssc)$ and $\Omega_b(ssb)$, from the current in Eq.(1), since one can not construct an scalar or pseudoscalar diquark in a $\bar{3}$ configuration of color, with two strange quarks. Therefore, to study the baryon Ω_Q we can use a proton-like current:

$$\eta_\Omega = \epsilon_{abc}(s_a^T C \gamma_\mu s_b) \gamma_5 \gamma^\mu Q_c. \quad (16)$$

With this current, the OPE contributions to the sum rule in the structure \not{q} (Eq. (9)) for the Ω_Q baryon, up to dimension-six condensates, are given by:

$$\begin{aligned} \rho_2^{pert}(s) &= \frac{m_Q^4}{2^6 \pi^4} \left[(1-x^2) \left(\frac{1}{x^2} - \frac{8}{x} + 1 \right) - 12 \ln x \right], \\ \rho_2^{\langle \bar{q}q \rangle}(s) &= 0, \\ \Pi_2^{\langle G^2 \rangle}(M^2) &= -\frac{m_Q^2 \langle g^2 G^2 \rangle}{2^{16} 3 \pi^4}, \int_0^1 d\alpha \frac{\alpha^2}{(1-\alpha)^2} e^{-m_Q^2/M^2}, \\ \Pi_2^{\langle \bar{q}g\sigma.Gq \rangle}(M^2) &= -\frac{7m_s}{48\pi^2} \langle \bar{s}g\sigma.Gs \rangle e^{-m_Q^2/M^2}, \\ \Pi_2^{\langle \bar{q}q \rangle^2}(M^2) &= \frac{2\mathcal{B}^2}{3} e^{-m_Q^2/M^2}. \end{aligned} \quad (17)$$

We find that the OPE convergence of this sum rule is as good as the OPE convergence obtained for Ξ_Q in the same Borel region: $M^2 \geq 1.5 \text{ GeV}^2$ for Ω_c and $M^2 \geq 4.0 \text{ GeV}^2$ for Ω_b . In Table II we give the maximum value of M^2 for which the continuum contribution is smaller than 50%, for different values of the continuum threshold.

Table II: Maximum values of the Borel parameter (M^2) for different values of s_0 .

state	$\sqrt{s_0}(\text{GeV})$	$M_{max}^2(\text{GeV}^2)$
Ω_c	3.2	2.7
Ω_c	3.4	3.1
Ω_b	6.4	5.5
Ω_b	6.7	6.5

The results obtained for m_{Ω_Q} , from Eq.(13), are also very stable, as a function of M^2 , in the allowed Borel window. Considering the variations in s_0 and M^2 given in Table II, and the variations in the quark masses and condensates, as discussed above, we get

$$m_{\Omega_c} = (2.65 \pm 0.25) \text{ GeV}, \quad (18)$$

in a very good agreement with the experimental value $m_{\Omega_c}^{exp} = (2.6975 \pm 0.0026) \text{ GeV}$ [7]. For Ω_b we make the prediction:

$$m_{\Omega_b} = (5.82 \pm 0.23) \text{ GeV}. \quad (19)$$

As a final remark, it is very reassuring to see that the OPE convergence is so good for heavy baryons, since this is not the case for tetraquark states [9]. As shown in ref. [9], it is very difficult to find a Borel region where the continuum contribution is bigger than the pole contribution and where the OPE convergence is acceptable, for tetraquark states with only one heavy quark. Therefore, from a QCD sum rule point of view, it is much easier to form an state, separated from the continuum, with three quarks than with four quarks.

Acknowledgements

This work has been partly supported by FAPESP and CNPq-Brazil.

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